

Lévy NMF for robust nonnegative source separation

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Abstract—Source separation, which consists in decomposing data into meaningful structured components, is an active research topic in many areas, such as music and image signal processing, applied physics and text mining. In this paper, we introduce the Positive α -stable (P α S) distributions to model the latent sources, which are a subclass of the stable distributions family. They notably permit us to model random variables that are both nonnegative and impulsive. Considering the Lévy distribution, the only P α S distribution whose density is tractable, we propose a mixture model called Lévy Nonnegative Matrix Factorization (Lévy NMF). This model accounts for low-rank structures in nonnegative data that possibly has high variability or is corrupted by very adverse noise. The model parameters are estimated in a maximum-likelihood sense. We also derive an estimator of the sources given the parameters, which extends the validity of the generalized Wiener filtering to the P α S case. Experiments on synthetic data show that Lévy NMF compares favorably with state-of-the-art techniques in terms of robustness to impulsive noise. The analysis of two types of realistic signals is also considered: musical spectrograms and fluorescence spectra of chemical species. The results highlight the potential of the Lévy NMF model for decomposing nonnegative data.

Index Terms—Lévy distribution, Positive alpha-stable distribution, nonnegative matrix factorization, source separation.

I. INTRODUCTION

SOURCE separation consists in extracting underlying components called *sources* that add up to form an observable signal called *mixture*. This issue occurs in various fields such as data mining [1], face recognition [2] or applied physics [3].

A groundbreaking idea presented in [4] is to exploit the fact that the observations are often nonnegative, so that they should be decomposed as a sum of only nonnegative terms. This Nonnegative Matrix Factorization (NMF) has shown successful in many fields such as audio signal processing [5], computer vision [4], spectroscopy [6] and many others.

NMF, originally introduced as a rank-reduction method, approximates a nonnegative data matrix X as a product of two low-rank nonnegative matrices W and H . The factorization can be obtained by optimizing a cost function measuring the error between X and WH , such as the Euclidean, Kullback-Leibler (KL [4]) and Itakura-Saito (IS [7]) cost functions. This may often be framed in a probabilistic framework, where the cost function appears as the negative log-likelihood of the data, e.g. Gaussian [7], [8] or Poisson [9], [10]. Addressing the estimation problem in a Maximum A Posteriori (MAP) sense makes it possible to incorporate some prior distribution over the parameters W and H . This allows some regularization

schemes enforcing desirable properties for the parameters such as harmonicity, temporal smoothness [11] or sparsity [12].

However, the above-mentioned distributions fail to provide good results when the data is very impulsive or contains outliers. This comes from their rapidly decaying tails, that cannot account for really unexpected observations. The family of heavy-tailed *stable* distributions [13] was thus found useful for robust signal processing [14], [15]. A subclass of this family, called the Symmetric α -stable (S α S) distributions, has been used in audio [16], [17] for modeling complex Short-Term Fourier Transforms (STFT). An estimation framework based on the Markov Chain Monte Carlo (MCMC) method has been proposed [18] to perform the separation of S α S mixtures. The common ground of these methods is to assume all observations as independent but to impose low-rank constraints on the nonnegative *dispersion parameters* of the sources, and not on their actual outcomes.

In this paper, we address the problem of modeling and separating nonnegative sources from their mixture, while still constraining their dispersion to follow an NMF model. To do so, we consider another subclass of the stable family which models nonnegative random variables: the positive α -stable (P α S) distributions. They also benefit from being heavy-tailed and are thus expected to yield robust estimates. Since the Probability Density Function (PDF) of those P α S distributions does not admit a closed-form expression in general, we study more specifically the Lévy case, which is a particular analytically tractable member of the family. We introduce the *Lévy NMF* model, where the dispersion parameters of the sources are structured through an NMF model and where realizations are necessarily nonnegative. The parameters are then estimated in a Maximum Likelihood (ML) sense by means of a Majorize-Minimization approach. We also derive an estimator of the sources which extends the validity of the generalized Wiener filtering to the P α S case. Several experiments conducted on synthetic, audio and fluorescence spectroscopy signals show the potential of this model for a nonnegative source separation task and highlight its robustness to impulsive noise.

This paper is structured as follows. Section II introduces the Lévy NMF mixture model. Section III details the parameters estimation and presents an estimator of the sources. Section IV experimentally demonstrates the denoising ability of the model and its potential in terms of source separation for both musical and chemometric applications.

II. LÉVY NMF MODEL

A. Positive α -stable distributions

Stable distributions, denoted $\mathcal{S}(\alpha, \mu, \sigma, \beta)$, are heavy-tailed distributions parametrized by four parameters: a shape parameter $\alpha \in]0; 2]$ which determines the tails thickness of the

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distribution (the smaller α , the heavier the tail of the PDF), a location parameter $\mu \in \mathbb{R}$, a scale parameter $\sigma \in]0; +\infty[$ measuring the dispersion of the distribution around its mode, and a skewness parameter $\beta \in [-1; 1]$. The symmetric α -stable (S α S) distributions, which are such that $\beta = 0$, are an important subclass of the stable family and a growing topic of interest, notably in audio [16], [18].

Such distributions are said "stable" because of their additive property: a sum of K independent stable random variables $X_k \sim \mathcal{S}(\alpha, \mu_k, \sigma_k, \beta)$ is also a stable random variable: $X = \sum_k X_k \sim \mathcal{S}(\alpha, \mu, \sigma, \beta)$, with $\mu = \sum_k \mu_k$ and $\sigma^\alpha = \sum_k \sigma_k^\alpha$.

Stable distributions do not in general have a nonnegative support. However, it can be shown [19] that when $\beta = 1$ and $\alpha < 1$, the support of the distribution is $[\mu; +\infty[$. In this paper, we consider that $\mu = 0$ thus the support is \mathbb{R}_+ . The Positive α -stable distributions are therefore such that $\mathcal{P}\alpha\mathcal{S}(\sigma) = \mathcal{S}(\alpha, 0, \sigma, 1)$ with $\alpha < 1$.

B. Lévy NMF mixture model

The only α for which the PDF of a P α S distribution can be expressed in closed form is the Lévy case $\mathcal{L}(\sigma) = P_{\frac{1}{2}}S(\sigma)$:

$$p(x | \sigma) = \begin{cases} \sqrt{\frac{\sigma}{2\pi}} \frac{1}{x^{3/2}} e^{-\frac{\sigma}{2x}} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

We model all the entries $X(f, t)$ of a data matrix $X \in \mathbb{R}_+^{F \times T}$ as independent, and assume they are the sum of K independent Lévy-distributed components $X_k(f, t) \sim \mathcal{L}(\sigma_k(f, t))$. Note that all entries are independent, which allows us to extend the standard notation to matrices. Then, $X \sim \mathcal{L}(\sigma)$ with $\sigma^{\odot 1/2} = \sum_k \sigma_k^{\odot 1/2}$, where \odot denotes the element-wise power.

The scale parameters are structured by means of an NMF [4], which preserves the additive property of the model as in [17]: $\sigma^{\odot 1/2} = WH$, with $W \in \mathbb{R}_+^{F \times K}$ and $H \in \mathbb{R}_+^{K \times T}$. We then refer to this model as the *Lévy NMF* model.

III. PARAMETERS ESTIMATION

The parameters $\theta = \{W, H\}$ are estimated in a Maximum Likelihood (ML) sense, which is natural in a probabilistic framework. The log-likelihood of the data is given by:

$$\begin{aligned} L(W, H) &= \sum_{f,t} \log(p(X(f, t); \sigma(f, t))) \\ &\stackrel{c}{=} \frac{1}{2} \sum_{f,t} \log([WH](f, t)^2) - \frac{[WH](f, t)^2}{X(f, t)} \\ &\stackrel{c}{=} -\frac{1}{2} d_{IS}([WH]^{\odot 2}, X), \end{aligned}$$

where $\stackrel{c}{=}$ denotes equality up to an additive constant which does not depend on the parameters and d_{IS} denotes the IS divergence [7]. We thus remark that maximizing the log-likelihood of the data in the Lévy NMF model is equivalent to minimizing the IS divergence between $[WH]^{\odot 2}$ and X , which boils down to minimizing the following cost function:

$$\mathcal{C}(W, H) = \sum_{f,t} \frac{[WH](f, t)^2}{X(f, t)} - 2 \log([WH](f, t)). \quad (2)$$

A. Naive multiplicative updates

The cost function (2) can be minimized with the same heuristic approach that has been pioneered in [4] and used in many NMF-related papers in the literature. The gradient of \mathcal{C} with respect to a parameter θ (W or H) is expressed as the difference between two nonnegative terms: $\nabla_{\theta} \mathcal{C} = \nabla_{\theta}^+ \mathcal{C} - \nabla_{\theta}^- \mathcal{C}$, which leads to the multiplicative update rules (MUR):

$$\theta \leftarrow \theta \odot \frac{\nabla_{\theta}^-}{\nabla_{\theta}^+} = \theta \odot a_{\theta}, \quad (3)$$

where \odot (resp. the fraction bar) denotes the element-wise matrix multiplication (resp. division). For Lévy NMF:

$$a_W = \frac{[WH]^{\odot -1} H^T}{([WH] \odot X^{\odot -1}) H^T}, \quad (4)$$

and

$$a_H = \frac{W^T [WH]^{\odot -1}}{W^T ([WH] \odot X^{\odot -1})}. \quad (5)$$

Provided W and H have been initialized as nonnegative, they remain so throughout iterations. However, these updates do not guarantee a non-increasing cost function \mathcal{C} , which motivates the research for a novel optimization approach.

B. Majorize-Minimization updates

An alternative way to derive update rules for estimating the parameters is to adopt a Majorize-Minimization (MM) approach [20]. The core idea of this strategy is to find an auxiliary function G which majorizes the cost function \mathcal{C} :

$$\forall(\theta, \bar{\theta}), \mathcal{C}(\theta) \leq G(\theta, \bar{\theta}), \text{ and } \mathcal{C}(\bar{\theta}) = G(\bar{\theta}, \bar{\theta}). \quad (6)$$

Then, given some current parameter $\bar{\theta}$, we aim at minimizing $G(\theta, \bar{\theta})$ in order to obtain a new parameter θ . This approach guarantees that the cost function \mathcal{C} will be non-increasing over iterations. Such an auxiliary function is obtained in a similar fashion as in [21], [22]. We have:

$$[WH](f, t)^2 = \left(\sum_k \rho_k(f, t) \frac{W(f, k) H(k, t)}{\rho_k(f, t)} \right)^2, \quad (7)$$

where

$$\rho_k(f, t) = \frac{\bar{W}(f, k) H(k, t)}{\bar{V}(f, t)}, \quad (8)$$

$\bar{V} = \bar{W}H$, and \bar{W} is an auxiliary parameter. Since $\sum_k \rho_k(f, t) = 1$, we can apply the Jensen's inequality to the convex function $(\cdot)^2$:

$$[WH](f, t)^2 \leq \sum_k \rho_k(f, t) \left(\frac{W(f, k) H(k, t)}{\rho_k(f, t)} \right)^2, \quad (9)$$

which finally leads to the following majorization of the first term of \mathcal{C} in (2):

$$\sum_{f,t} \frac{[WH](f, t)^2}{X(f, t)} \leq \sum_{f,t} \frac{\bar{V}(f, t)}{X(f, t)} \sum_k \frac{W(f, k)^2 H(k, t)}{\bar{W}(f, k)}. \quad (10)$$

In a similar fashion, we majorize the second term in (2) ($-\log(\cdot)$ is also a convex function) and therefore we obtain

the following auxiliary function G :

$$G(W, H, \bar{W}) = \sum_{f,t,k} \frac{\bar{V}(f,t)H(k,t)}{X(f,t)\bar{W}(f,k)} W(f,k)^2 - 2 \frac{\bar{W}(f,k)H(k,t)}{\bar{V}(f,t)} \log \left(W(f,k) \frac{\bar{V}(f,t)}{\bar{W}(f,k)} \right). \quad (11)$$

We then set the partial derivative of G with respect to W to zero, which leads to an update rule on W . The update rule on H is obtained in exactly the same way. Thus, the updates are:

$$\theta \leftarrow \theta \odot a_\theta^{\odot 1/2}, \quad (12)$$

where a_W and a_H are given by (4) and (5). A MATLAB implementation of this algorithm can be found at [23].

C. Estimator of the components

For a source separation task, it can be useful, once the model parameters are estimated, to derive an estimator \hat{X}_k of the isolated components X_k . In a probabilistic framework, a natural estimator is given by the posterior expectation of the source given the mixture $\mathbb{E}(X_k|X)$. It has been shown [16] that for $S\alpha S$ random variables, such an estimator is provided by a generalized Wiener filtering. One contribution of this paper is to prove that this result still holds for the $P\alpha S$ family. This finding extends α -Wiener filter for the separation of nonnegative variables. Derivations may be found in the companion technical report for this paper [24]. For Lévy-distributed random variables ($\alpha = 1/2$), we then have:

$$\hat{X}_k = \mathbb{E}(X_k|X) = \frac{\sigma_k^{\odot 1/2}}{\sum_l \sigma_l^{\odot 1/2}} \odot X = \frac{W_k H_k}{\sum_l W_l H_l} \odot X. \quad (13)$$

IV. EXPERIMENTAL EVALUATION

A. Fitting impulsive noise

To test the ability of Lévy NMF to model impulsive noise, we have generated 5 components' pairs for W and H by taking the 4th power of random Gaussian noise, in order to obtain sparse components. The entries of the product $[WH]$, of dimensions 50×50 , were then used as the scale parameters of independent $P\alpha S$ random observations, for various values of α in the range $0.1 - 0.5$: small values of α lead to very impulsive observations. We ran 200 iterations of the Lévy, IS [7], KL [4] and Cauchy [17] NMFs, and RPCA [25], [26] algorithms with rank $K = 5$. To measure the quality of the estimation, we computed the KL divergence and the α -dispersion, defined as a function of the data shape parameter α :

$$L_\alpha = \sum_{f,t} |\sigma(f,t) - \hat{\sigma}(f,t)|^{1/\alpha}, \quad (14)$$

where $\sigma^{\odot \alpha} = WH$ contains the synthetic parameters and $\hat{\sigma}^{\odot \alpha} = \hat{W}\hat{H}$ contains the parameters estimated with the different algorithms. Results averaged over 100 synthetic data runs are presented in Fig. 1. We observe that the Lévy NMF algorithm shows very similar results to those obtained using RPCA or Cauchy NMF, with slightly better results than these methods for very small values of α . The reconstruction quality

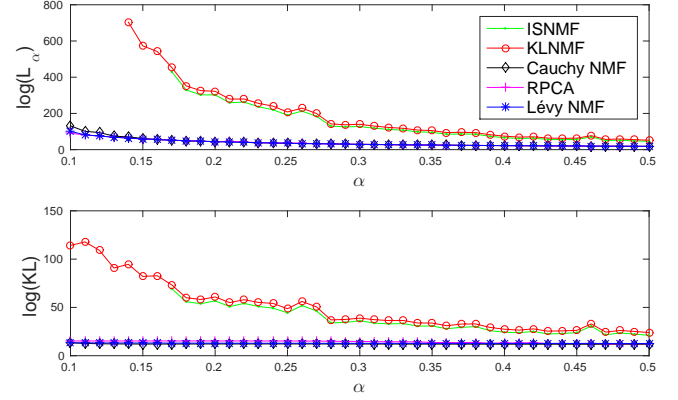


Fig. 1. Fitting impulsive noise by different algorithms, measured with the α -dispersion and KL divergence.

is considerably better than with ISNMF and KLNMF. Those results demonstrate the potential of the Lévy NMF model for fitting data which may be very impulsive.

B. Music spectrogram inpainting

We propose to test the denoising ability of the Lévy NMF model when the data is corrupted by very impulsive noise. When audio spectrograms are corrupted by such noise, the retrieval of the lost information is known as an *audio inpainting* task. We consider 6 guitar songs from the IDMT-SMT-GUITAR [27] database, sampled at 8000 Hz. The data X is obtained by taking the magnitude spectrogram of the STFT of the mixture signals, computed with a 125 ms-long Hann window and 75 % overlap. The spectrograms are then corrupted with synthetic impulsive noise that represents 10 % of the data. We then run 200 iterations of the algorithms with rank $K = 30$ in order to estimate the clean spectrograms.

We present the obtained spectrograms in Fig. 2 (KLNMF and ISNMF lead to similar results). It appears that the traditional NMF techniques are not able to denoise the data: the estimation of the parameters is deteriorated by the presence of impulsive noise. Conversely, the noise has been entirely removed in the Lévy NMF estimate. This is confirmed in Table I which presents the quality of the estimation measured with the KL divergence between the original and estimated spectrograms averaged over the 6 audio excerpts. The best results are obtained with Lévy and Cauchy NMFs.

Remarkably, none of the algorithms used above is informed with any prior knowledge about the location of the noise. As a complementary experiment, we informed ISNMF with this knowledge by incorporating a mask containing the position of the noise into the NMF, resulting into a weighted ISNMF [28]. The blind Lévy NMF still leads to better results than the informed ISNMF. It is thus promising for robust musical applications such as audio inpainting, since it does not require any additional noise modeling or detection technique.

C. Application to fluorescence spectroscopy

Fluorescence spectroscopy [6] consists in measuring the excitation-emission spectra of a mixture, which is modeled

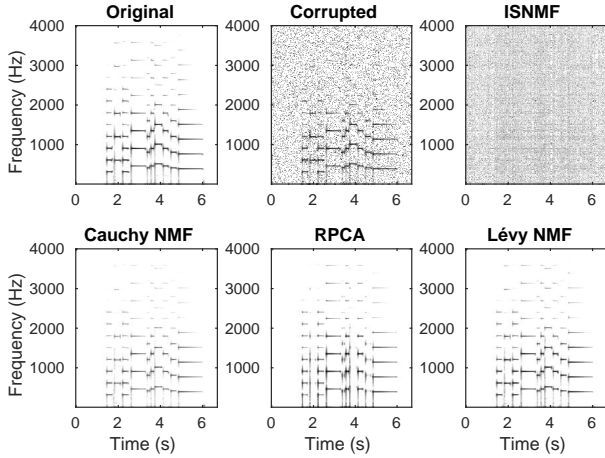


Fig. 2. Music spectrogram restoration.

TABLE I
AVERAGE LOG-KL DIVERGENCE BETWEEN CLEAN AND ESTIMATED
MUSIC SPECTROGRAMS.

ISNMF	9.0
KLNMF	6.2
Cauchy NMF	3.4
RPCA	3.6
Lévy NMF	3.2
Weighted ISNMF	3.8

as a the weighted sum of the spectra of the pure species. Performing source separation on these data allows us to identify the pure species composing the mixture and their concentrations. NMF-based methods have shown promising results for this task [29], since the data is nonnegative and the NMF model conforms to the underlying assumption of additivity of pure spectra. In this framework, W represents the spectra and H is the concentration matrix.

We propose to apply the Lévy NMF model to such a task. We compare it with the NMF with Euclidean distance (EuNMF) [30] and with KL divergence [31]. Indeed, other techniques (ISNMF, Cauchy NMF and RPCA) focus on the modeling of complex-valued data, or do not enforce a non-negative property of the parameters. Thus, it would not be theoretically justified to use those methods in this context.

Each algorithm uses 50 iterations of MUR. The dataset is detailed in [29]. In a nutshell, it consists of $T = 400$ emission spectra (with $F = 128$ frequency channels) of mixtures of $K = 3$ components (bound ferulic acid, free ferulic acid and p-coumaric acid) with unknown concentrations. Both W and H are learned directly from the mixtures. We also have access to the pure spectra of the components. As it is shown in Fig. 3, the Lévy NMF model seems to be an appropriate tool to learn pure fluorescence spectra from their mixtures. Globally, both Euclidean, KL and Lévy NMF approximate quite accurately the original pure spectra, though the Lévy NMF estimate seems to approach the spectra from above.

Finally, we estimate the isolated sources by means of (13) for the different methods. As a comparison reference, we also

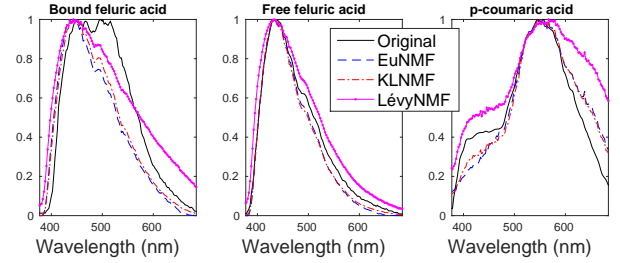


Fig. 3. Pure components' spectra learned with several NMF methods.

TABLE II
CORRELATION (IN %) BETWEEN ORACLE AND ESTIMATED SOURCES.

	EuNMF	KLNMF	Lévy NMF
Bound ferulic acid	82.8	85.3	87.8
Free ferulic acid	99.5	99.4	99.6
p-coumaric acid	97.3	98.1	98.4

learn the concentration matrix H when assuming the pure spectra are known (Oracle case) by means of EuNMF (results obtained with other NMFs are similar). The similarity between the Oracle sources and the estimated sources is measured by means of the correlation, the values of which are presented in Table II. The best performance is obtained with the Lévy NMF method for all sources, which confirms the potential of such a model in various areas of research involving source separation of nonnegative data.

V. CONCLUSION

In this paper, we introduced the Lévy NMF model, which structures the dispersion parameters of $P\alpha S$ distributed sources when $\alpha = 1/2$. Experiments have shown the potential of this model for robustly decomposing realistic nonnegative data.

Such a model could be useful in many other fields where the source separation issue frequently occurs and where the Lévy distribution finds applications, such as optics [32] or geomagnetic field analysis [33]. Future work could focus on novel estimation techniques for the Lévy NMF model, using for instance a MAP estimator, which would permit us to incorporate some prior knowledge about the parameters [11]. Besides, drawing on [18], the family of techniques based on MCMC could be useful to estimate the parameters of any $P\alpha S$ distribution. Alternatively, one could extend the Lévy NMF model to the family of inverted gamma (IG) of which it is a special case. Although it would be losing additivity and thus the theoretical foundation for α -Wiener filtering, this would allow for convenient analytical derivations thanks to tractable likelihood functions. This strategy is reminiscent of recent work [34] where the tractable student-t distribution is used instead of the symmetric α -stable one.

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